PERMUTATIONS & COMBINATIONS

(KEY CONCEPTS + SOLVED EXAMPLES)

PERMUTATIONS & COMBINATIONS

KEY CONCEPTS

1. Fundamental Principles of Operation

When one or more operations can be accomplished by number of ways then there are two principles to find the total number of ways to accomplish one, two, or all of the operations without counting them as follows:

1.1 Fundamental Principle of Multiplication :

Let there are two parts A and B of an operation and if these two parts can be performed in m and n different number of ways respectively, then that operation can be completed in $m \times n$ ways.

1.2 Fundamental Principle of addition :

If there are two operations such that they can be done independently in m and n ways respectively, then any one of these two operations can be done by $(m + n)$ number of ways.

2. Combinations

The different groups or selections of a given number of things by taking some or all at a time without paying any regard to their order, are called their **combinations.**

The number of combinations of n **<u>different</u>** things taken r at a time is denoted by

$${}^nC_r \text{ or } C(n, r)$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

So $${}^nC_r = \frac{n(n-1)(n-2).....(n-r+1)}{r!}$$

Particular cases : $${}^nC_r = \frac{{}^np_r}{r!}$$

$${}^nC_n = 1$$

$${}^nC_0 = 1$$

Some Important Results :

* ${}^nC_r = {}^nC_{n-r}$
* ${}^nC_x = {}^nC_y \Rightarrow x + y = n$
* ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
* ${}^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1}$
* ${}^nC_r = \frac{1}{r}(n-r+1)\,{}^nC_{r-1}$
* ${}^nC_1 = {}^nC_{n-1} = n$
* Greatest value of ${}^nC_r$
  = ${}^nC_{n/2}$, when n is even
  = ${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$, when n is odd

2.1 Restricted Combinations :

The number of combinations of n different things taking r at a time

(a) When p particular things are always to be included = $\mathbf{{}^{n-p}C_{r-p}}$

(b) When p particular things are always to be excluded = $\mathbf{{}^{n-p}C_r}$

(c) When p particular things are always included and q particular things are always excluded

= $\mathbf{{}^{n-p-q}C_{r-p}}$

2.2 Total number of combinations in different cases :

(a) The number of combinations of n **<u>different things</u>** taking some or all (or atleast one) at a time $= {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$= \mathbf{2^n - 1}$

(b) The number of ways to select some or all out of (p + q + r) things where p are alike of first kind, q are alike of second kind and r are alike of third kind is = **(p +1) (q +1) (r +1) – 1**

(c) The number of ways to select some or all out of (p + q + t) things where p are alike of first kind, q are alike of second kind and remaining t are different is = **(p +1) (q +1) 2^t – 1**

3. Permutations

An arrangement of some given things taking some or all of them, is called a **permutation** of these things.

For Example, three different things a, b and c are given, then different arrangements which can be made by taking two things from the three given things are

ab, ac, bc, ba, ca, cb

Therefore, the number of permutations will be 6.

3.1 The number of permutations of n **<u>different</u>** things taken r at a time is ${}^nP_r$, where

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$= n\,(n-1)\,(n-2)\,.....\,(n-r+1)$$

The number of permutations of n dissimilar things taken all at a time = ${}^nP_n = n!$

3.2 Permutations in which all things are not different :

The number of permutations of n things taken **<u>all at a time</u>** when p of them are alike and of one kind, q of them are alike and of second kind, r of them are alike and of third kind and all remaining being different is

$$\frac{n!}{p!q!r!}$$

3.3 Permutations in which things may be repeated :

The number of permutations of n **<u>different things</u>** taken r at a time when each thing can be used once, twice, ...upto r times in any permutation is $\mathbf{n^r}$.

In particular, in above case when n things are taken at a time then total number of permutation is $\mathbf{n^n}$.

3.4 Restricted Permutations : If in a permutation, some particular things are to be placed at some particular places or some particular things are always to be included or excluded, then it is called a **restricted permutation.** The following are some of the restricted permutations.

(a) The number of permutations of n dissimilar things taken r at a time, when m particular things always occupy definite places = $\mathbf{{}^{n-m}P_{r-m}}$

(b) The number of permutations of n different things taken altogether when r particular things are to be placed at some **r** given places.

$= \mathbf{{}^{n-r}P_{n-r} = (n-r)\,!}$

(c) The number of permutations of n different things taken r at a time, when m particular things are always to be excluded = $\mathbf{{}^{n-m}P_r}$

(d) The number of permutations of n different things taken r at a time when m particular things are always to be included

$= \mathbf{{}^{n-m}C_{r-m}} \times \mathbf{r!}$

3.5 Permutation of numbers when given digits include zero :

If the given digits include 0, then two or more digit numbers formed with these digits cannot have 0 on the extreme left. In such cases we find the number of permutations in the following two ways.

(a) (The number of digits which may be used at the extreme left) x (The number of ways in which the remaining places may be filled up)

(b) If given digits be n (including 0) then total number of m- digit numbers formed with them

$$= {}^{n}P_{m} - {}^{n-1}P_{m-1}.$$

because ${}^{n-1}P_{m-1}$ is the number of such numbers which contain 0 at extreme left.

3.6 Circular Permutations

Till now we have calculated the number of linear permutation in which things are arranged in a row. Now we shall find the number of permutations in which things are arranged in a circular shape. Such permutations are named as **circular permutations**. Thus an arrangement of some given things round a circle is called their circular permutation.

It should be noted that in a circular permutation initial and final position of things can not be specified. Thus all linear permutations of some given things having the same order of elements will give the same circular permutation.

For example, there are 6 linear permutations of three letters A, B and C taken all at a time. These are ABC, ACB, BAC, BCA, CAB, CBA

(Anti- clockwise order) (Clockwise order)

Since the arrangements ABC, BCA, CAB, are in the same order (clockwise order), therefore these three linear permutations are equal to one circular permutation.

From this example, it is clear that from a circular permutation of three things, there correspond three linear permutations. Thus, we conclude that if x be the number of circular permutations of 3 given things then the number of their linear permutations will be 3x.so

$$3x = 3! \Rightarrow x = \frac{3!}{3}.$$

In a similar way it can be seen that if x be the number of circular permutations of n different things taking r at a time, then

$$rx = {}^{n}P_{r} \Rightarrow x = {}^{n}P_{r}/r$$

Thus, we obtain the following results for the number of circular permutations.

3.6.1 Number of Circular Permutations :

(a) The number of Circular permutations of n **<u>different</u>** things taking r at a time $\frac{{}^{n}P_{r}}{r}$, when clockwise and anti-clockwise orders are treated as different.

(b) The number of circular permutations of n **<u>different</u>** things taking altogether

$\frac{{}^{n}P_{n}}{n}$, when clockwise and anti clockwise orders are treated as different.

(c) The number of Circular permutations of n **<u>different</u>** things taking r at a time $\frac{{}^{n}P_{r}}{2r}$, when the above two orders are treated as same.

(d) The number of circular permutations of n **<u>different</u>** things taking altogether

$\frac{{}^{n}P_{n}}{2n} = \frac{1}{2}(n-1)!$, when above two orders are treated as same.

3.6.2 Restricted Circular Permutations : When there is a restriction in a Circular permutation then first of all we shall perform the restricted part of the operation and then perform the remaining part treating it similar to a linear permutation.

4. Division into Groups

(a) The number of ways in which (p + q) things can be divided into two groups of p and q things is

$${}^{p+q}C_{p} = {}^{p+q}C_{q} = \frac{(p+q)!}{p!\,q!}$$

Particular case : when p = q, then total number of combinations are

(i) $\frac{2p\,!}{(p!)^2}$ when groups are differentiable.

(ii) $\frac{2p\,!}{2!\,(p!)^2}$ when groups are not differentiable.

(b) The number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things is

$$\frac{(p+q+r)!}{p!\,q!\,r!}$$

Particular case :

when p = q = r, then total number of combinations are

(i) $\frac{3p!}{(p!)^3}$ when groups are differentiable.

(ii) $\frac{3p!}{3!\,(p!)^3}$ when groups are not differentiable.

5. Permutations in Which the Operation of Selection is Necessary

There are questions of permutation in which we have to start with the operation of selection for the given number of things. After this we calculate the number of different arrangements for each of such selected group.

6. Dearrangement Theorem

Any change in the given order of the things is called a **Dearrangement** .

(a) If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is $n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....(-1)^n\frac{1}{n!}\right]$

(b) If n things are arranged at n places then the number of ways to rearrange exactly r things at right places is

$$\frac{n!}{r!}\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....(-1)^{n-r}\frac{1}{(n-r)!}\right]$$

7. Multinomial Theorem & Its Applications

7.1 Multinomial Theorem :

The expansion of $[x_1 + x_2 + x_3 + ... + x_n]^r$ where n & r are integers ($0 < r \le n$) is a homogenous expression in x_1, x_2, x_3, ... x_n and given as below :

$[x_1 + x_2 + x_3 + ... + x_n]^r$

$$= \sum\left(\frac{r!}{\lambda_1!\lambda_2!\lambda_3!....\lambda_n!}\right)x_1^{\lambda_2}x_2^{\lambda_2}x_3^{\lambda_3}....x_n^{\lambda_n}$$

(where n & r are integers $0 \le r \le n$ and

λ_1, λ_2, ..., λ_n are non negative integers)

Such that $\lambda_1 + \lambda_2 + ... + \lambda_n = r$

(valid only if x_1, x_2, x_3, ... x_n are independent of each other)

coefficient of $x_1^{\lambda_1}x_2^{\lambda_2}x_3^{\lambda_3}$.....= total number of arrangements of r objects out of which λ_1 number of x_1's are identical λ_2 number of x_2's are identical and so on .....

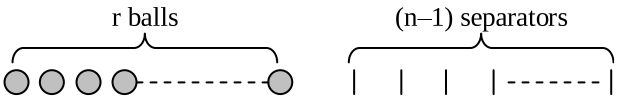
$$= \left(\frac{(\lambda_1 + \lambda_2 + \lambda_3 + ...\lambda_n)!}{\lambda_1! \lambda_2! \lambda_3! ....\lambda_4!} \right) = \frac{r!}{\lambda_1! \, \lambda_2! \, \lambda_3! ....\lambda_n!}$$

7.2 Number of distinct terms :

Since $(x_1 + x_2 + x_3 + ... + x_n)^r$ is multiplication of $(x_1 + x_2 + x_3 + ... + x_n)$, r times & will be a homogeneous expansion of r^{th} degree in $x_1, x_2, ...x_n$ So in each term sum of powers of variables must be r

So number of distinct terms will be total number of non-negative integral solution of equation is $\lambda_1 + \lambda_2 + \lambda_3 + ... + \lambda_n = r$

= Number of ways of distributing r identical objects among n persons



= number of arrangements of r identical balls & n – 1 identical separators.

$$= \frac{(n-1+r)!}{(n-1)!\, r!} = {}^{n+r-1}C_r = {}^{n+r-1}C_{n-1}$$

7.3 Application of multinomial theorem

If we want to distribute n identical objects in r different groups under the condition that empty groups are not allowed. $a_1 + a_2 + a_3 + ... + a_r = n$

Boundary conditions are $1 \le a_1, a_2......a_r \le n$

(As each box contains at least one object)

Number of ways

= coefficient of x^n in $(x^1 + x^2 + ... + x^n)^r$

= coefficient of x^{n-r} in $(1 + x + x^2 + ... + x^{n-1})^r$

$= {}^{(n-r)+r-1}C_{r-1} = {}^{n-1}C_{r-1}$

8. Divisibility of Numbers

The following chart shows the conditions of divisibility of numbers by 2,3,4,5,6,8,9,25

Divisible by	**Condition**	
2	whose last digit is even	(2, 4, 6, 8, 0)
3	sum of whose digits is	divisible by 3
4	whose last two digits number	is divisible by 4
5	whose last digit is either 0 or 5	
6	which is divisible by both 2 and 3	
8	whose last three digits number	is divisible by 8
9	sum of whose digits is	divisible by 9
25	whose last two digits are	divisible by 25

9. Sum of Numbers

(a) For given n different digits $a_1, a_2, a_3 ......, a_n$ the sum of the digits in the unit place of all numbers formed (if numbers are not repeated) is

$(a_1 + a_2 + a_3 + ... + a_n)\ (n-1)!$

i.e. (sum of the digits) (n-1)!

(b) Sum of the total numbers which can be formed with given n different digits $a_1, a_2, a_3 .....a_n$ is **$(a_1 + a_2 + a_3 + ... + a_n)(n-1)!.(111 ...n \text{ ties})$**

10. Some Important Results About Points

If there are n points in a plane of which m (< n) are collinear, then

(a) Total number of different straight lines obtained by joining these n points is

$$\mathbf{{}^{n}C_2 - {}^{m}C_2 + 1}$$

(b) Total number of different triangles formed by joining these n points is

$$\mathbf{{}^{n}C_3 - {}^{m}C_3}$$

(c) Number of diagonals in polygon of n sides is

$$\mathbf{{}^{n}C_2 - n} \text{ i.e. } \frac{n(n-3)}{2}$$

(d) If m parallel lines in a plane are intersected by a family of other n parallel lines. Then total number of parallelograms so formed is

$$\mathbf{{}^{m}C_2} \times \mathbf{{}^{n}C_2} \quad \text{i.e.} \quad \frac{mn(m-1)(n-1)}{4}$$

SOLVED EXAMPLES

Ex.1 If $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, then the value of x is-

(A) 123 (B) 125

(C) 121 (D) None of these

Sol. $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!} \Rightarrow \frac{1}{9!}+\frac{1}{10.9!}=\frac{x}{11.10.9!}$

$\Rightarrow \frac{1}{9!}\left[1+\frac{1}{10}\right]=\left(\frac{x}{11.10}\right).\frac{1}{9!}$

$\Rightarrow 1+\frac{1}{10}=\frac{x}{11.10}$

$\Rightarrow \frac{11}{10}=\frac{x}{11.10} \Rightarrow x = 11.11 = 121$

Ans. [C]

Ex.2 The number of different words (meaningful or meaningless) can be formed by taking four different letters from English alphabets is-

(A) $(26)^4$ (B) 358800

(C) $(25)^4$ (D) 15600

Sol. The first letter of four letter word can be chosen by 26 ways, second by 25 ways, third by 24 ways and fourth by 23 ways. So number of four letter words

$= 26 \times 25 \times 24 \times 23 = 358800$

Ans. [B]

Ex.3 If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$ then the value of **r** is -

(A) 14 (B) 41 (C) 51 (D) 10

Sol. $\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}}=\frac{30800}{1}$

$\Rightarrow \frac{56!}{(56-r-6)!}=\frac{(30800)\times 54!}{(54-r-3)!}$

$\Rightarrow 56 \times 55 \times (51-r) = 30800$

$\Rightarrow (51-r)=\frac{30800}{56\times 55} = 10$

$\Rightarrow r = 51 - 10 = 41$ **Ans. [B]**

Ex.4 The number of ways in which 2 vacancies can be filled up by 13 candidates is-

(A) 25 (B) 78 (C) 156 (D) 169

Sol. The no. of ways to fill up 2 vacancies by 13 candidates is-

${}^{13}P_2 = 13 \times 12 = 156$ **Ans. [C]**

Ex.5 How many different words beginning with A and ending with L can be formed by using the letters of the word' ANILMANGAL'?

(A) 10080 (B) 40320

(C) 20160 (D) None of these

Sol. After fixing the letters A and L in the first and last places, the total number of available places are 8 and the letters are also 8. Out of these 8 letters there are 2 groups of alike letters.

Therefore no. of words = $\frac{8!}{2!\,2!}$ =10080

Ans.[A]

Ex.6 How many numbers can be formed between 20000 and 30000 by using digits 2, 3, 5, 6, 9 when digits may be repeated?

(A) 125 (B) 24

(C) 625 (D) 1250

Sol. First digit between 20000 and 30000 will be 2 which can be chosen by one way. Every number will be of five digits and all the digits can be anything from the given five digit except first digit. So each digit of the remaining four digits can be chosen in 5 ways

$\therefore$ required numbers = $1 \times 5 \times 5 \times 5 \times 5 = 625$

Ans. [C]

Ex.7 The number of three letters words can be formed from the letters of word 'SACHIN' when I do not come in any word is-

(A) 120 (B) 60

(C) 24 (D) 48

Sol. There are 6 letters in the given word. Then the number of three letters words from the remaining 5 letters after removing I is-

$$= {}^5P_3 = 5 \times 4 \times 3 = 60$$

Ans. [B]

Ex.8 The number of numbers lying between 100 and 1000 which can be formed with the digits 0, 1, 2, 3, 4, 5, 6 is-

(A) 180 (B) 216

(C) 200 (D) None of these

Sol. Required numbers will have 3 digits so their total number = ${}^7P_3 - {}^6P_2 = 180$

Ans. [A]

Ex.9 How many numbers between 1000 and 4000 (including 4000) can be formed with the digits 0, 1, 2, 3, 4 if each digit can be repeated any number of times?

(A) 125 (B) 275

(C) 375 (D) 500

Sol. Required number will have 4 digits and their thousand digit will be 1 or 2 or 3 or 4. The number of such numbers will be 124, 125, 125 and 1 respectively.

$\therefore$ Total numbers = 124 + 125 + 125 + 1 = 375

Ans. [C]

Ex.10 The number of ways in which 7 girls can be stand in a circle so that they do not have the same neighbour in any two arrangements?

(A) 720 (B) 380

(C) 360 (D) None of these

Sol. Seven girls can keep stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle.

$\therefore \frac{(7-1)!}{2!} = 360$ **Ans. [C]**

Ex.11 The number of ways in which 7 men and 7 women can sit on a circular table so that no two women sit together is

(A) 7! . 7! (B) 7! . 6!

(C) $(6!)^2$ (D) 7!

Sol. Here one women will sit between two men. Now fixing the place of one man the remaining 6 men on the circular table can sit in 6! ways. Since there are seven places between 7 men. Therefore seven women can sit on these places in 7! ways.

Thus 7 men and 7 women under the given condition can sit in 7! **.** 6! ways.

Ans. [B]

Ex.12 ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$ is equal to -

(A) ${}^{51}C_4$ (B) ${}^{52}C_4$

(C) ${}^{53}C_4$ (D) None of these

Sol. The given expression can be written as

$$\sum_{r=1}^{5} {}^{52-r}C_3 + {}^{47}C_4$$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

[We know that ${}^{n+1}C_{r+1} = {}^nC_r + {}^nC_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$ **Ans. [B]**

Ex.13 A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. The number of ways in which he can make up his choice is-

(A) 100 (B) 200 (C) 300 (D) 400

Sol. Let there be two groups A and B each containing 5 questions. Questions to be attempted is 6, but not more than 4 from any group. The candidate can select the questions in following ways:

(i) 4 from group A and 2 from group B.

(ii) 3 from group A and 3 from group B.

(iii) 2 from group A and 4 from group B

The number of selections in the above cases are $^5C_4 \times {}^5C_2, {}^5C_3 \times {}^5C_3, {}^5C_2 \times {}^5C_4$ respectively.

$\therefore$ Number of ways of selecting 6 questions

$= {}^5C_4 \times {}^5C_2 + {}^5C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_4$

$= 50 + 100 + 50 = 200$ **Ans. [B]**

Ex.14 In how many ways can a committee consisting of one or more members be formed out of 12 members of the Municipal Corporation-

(A) 4095 (B) 5095 (C) 4905 (D) 4090

Sol. Required number of ways

$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + ... + {}^{12}C_{12} = 2^{12} - 1$

$= 4096 - 1 = 4095$ **Ans. [A]**

Ex.15 Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is-

(A) 881 (B) 891 (C) 879 (D) 892

Sol. The required number of ways are

$(10 + 1)(9 + 1)(7 + 1) - 1$

$= 11 \times 10 \times 8 - 1 = 879$

Ans. [C]

Ex.16 The number of words which can be formed taking 4 different letters out of the letters of the word 'ASSASSINATION', is-

(A) $^{13}C_4 . 4!$ (B) $^6C_4 . 4!$

(C) $^{13}P_4 / 2!$ (D) None of these

Sol. Total No. of selections of 4 different letters = $^6C_4$

$\therefore$ Total no. of different words = $^6C_4 . 4!$

Ans. [B]

Ex.17 There are four balls of different colours and four boxes of colours same as those of the balls. The number of ways in which the balls, one each box, could be placed such that a ball does not go to box of its own colour is-

(A) 8 (B) 7

(C) 9 (D) None of these

Sol. Number of derangements are

$$= 4!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\} = 12 - 4 + 1 = 9$$

{Since number of derangements in such a problems is given by n!

$$\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - (-1)^n \frac{1}{n!}\right\}$$

Ans. [C]

Ex.18 The number of 4 digit numbers divisible by 5 which can be formed by using the digits 0, 2, 3, 4, 5 is-

(A) 36 (B) 42

(C) 48 (D) None of these

Sol. For a number to be divisible by 5, unit place should be occupied by 0 or 5-

(i) If unit place is 0 then remaining 3 places can be filled by $^4P_3$ ways = 24

(ii) If unit place is 5 then no. of ways

$= 4! - 3! = 18$

$\therefore$ Total number of ways

$= 24 + 18 = 42$ **Ans. [B]**

Ex.19 The sum of all 5 digit numbers which can be formed using digits 1, 2, 3, 4, 5 is-

(A) 6666666 (B) 6600000

(C) 3999960 (D) None of these

Sol. Using formula (Here n = 5)

Sum = (1+ 2 + 3 + 4 + 5) 4! (11111)

$= 15 \times 24 \times 11111 = 3999960$

Ans. [C]

Ex.20 The number of diagonals in an octagon are -

(A) 28 (B) 48

(C) 20 (D) None of these

Sol. Here n = 8 (given)

The number of diagonals are given by

$$= \frac{n(n-3)}{2}$$

$$\Rightarrow \frac{8(8-3)}{2} = \frac{8.5}{2} = 20$$ **Ans. [C]**

Ex.21 To fill up 12 vacancies, there are 25 candidates of which 5 are from SC. If 3 of these vacancies are reserved for SC candidates while the remaining are open to all; then the number of ways in which the selection can be made is-

(A) $^5C_3 \times {}^{15}C_9$ (B) $^5C_3 \times {}^{22}C_9$

(C) $^5C_3 \times {}^{20}C_9$ (D) None of these

Sol. 3 vacancies for SC candidates can be filled up from 5 candidates in ${}^5C_3$ ways.

After this for remaining 12 – 3 = 9 vacancies, there will be 25 – 3 = 22 candidates. These vacancies can be filled up in ${}^{22}C_9$ ways.

Hence required number of ways = ${}^5C_3 \times {}^{22}C_9$

Ans. [B]

Ex.22 Out of 10 given points 6 are in a straight line. The number of the triangles formed by joining any three of them is-

(A) 100 (B) 150

(C) 120 (D) None of these

Sol. A triangle can be formed by joining three points, so there will be ${}^{10}C_3$ triangles joining any three out of 10 points. But 6 of these 10 points are collinear so these 6 points will give no triangle. Hence the required number of triangles $= {}^{10}C_3 - {}^6C_3$

$= 120 - 20 = 100$

Ans. [A]

Ex.23 The number of ways in which 5 biscuits can be distributed among two children is-

(A) 32 (B) 31

(C) 30 (D) None of these

Sol. Each biscuit can be distributed in 2 ways.

Therefore number of ways of distributing the biscuits

$= 2^5 = 32$

Now number of ways in which either of the two children does not get any biscuit = 2.

Required number of ways of distribution

$= 32 - 2 = 30$

Ans. [C]

Ex.24 How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word 'EQUATION' so that the two consonants occur together?

(A) 1380 (B) 1420

(C) 1440 (D) none

Sol. There are 5 vowels and 3 consonants in the word 'EQUATION'. Three vowels out of 5 and 2 consonants out of 3 can be chosen in ${}^5C_3 \times {}^3C_2$ ways. So, there are ${}^5C_3 \times {}^3C_2$ groups each containing two consonants and three vowels. Now, each group contains 5 letters which are to be arranged in such a way that 2 consonants occur together. Considering 2 consonants as one letter, we have 4 letters which can be arranged in 4! ways. But two consonants can be put together in 2! ways. Therefore, 5 letters in each group can be arranged in 4! × 2! ways.

Hence, the required number of words =

$({}^5C_3 \times {}^3C_2) \times 4! \times 2! = 1440$ **Ans. [C]**

Ex.25 Find the total number of proper factors of 7875.

(A) 20 (B) 22

(C) 24 (D) None of these

Sol. We have : $7875 = 3^2 \times 5^3 \times 7^1$

The total number of ways of selecting some or all out of two 3's, three 5's and one 7's is

$(2+1)(3+1)(1+1) - 1 = 23$

But this includes the given number itself. Therefore, the required number of proper factors is 22. **Ans. [B]**

Ex.26 The number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is-

(A) ${}^7P_2 .2^5$ (B) ${}^7C_2 .2^5$

(C) ${}^7C_2 .5^2$ (D) None of these

Sol. Choose any two of the seven digits (in the seven digit number). This may be done in ${}^7C_2$ ways. Put 2 in these two digits. The remaining 5 digits may be arranged using 1 and 3 in 2^5 ways. So, required number of numbers = ${}^7C_2 \times 2^5$.

Ans. [B]

Ex.27 How many numbers can be formed with the digits 0,1,2, 3,4,5 which are greater than 3000?

(A) 180 (B) 360

(C) 1380 (D) 1500

Sol. The numbers greater than 3000 may contain 4, 5 or 6 digits. Now the 6 digit numbers can be formed by using all 6 given digits which will be

$^6P_6$. These numbers include the numbers which starts with 0. Such type of numbers are $^5P_5$.

Therefore 6 digit numbers greater than 3000

$= {^6P_6} - {^5P_5} = 600$

Similarly 5 digit numbers greater than 3000

$= {^6P_5} - {^5P_4} = 600$

Now 4 digit numbers which are greater than 3000 should begin with the digit 3,4 or 5. If the first place of the number is occupied by the digit 3, 4 or 5, then the remaining three places of the number can be filled in $^5P_3 = 60$ ways.

Therefore numbers of 4 digits greater than 3000 = 3 x 60 = 180

Hence required numbers = 600 + 600 + 180 = 1380 **Ans. [C]**

Ex.28 In how many ways the letters AAAAA, BBB, CCC, D, EE, F can be arranged in a row when the letter C occur at different places?

(A) $\dfrac{12!}{5!\,3!\,2!} \times {^{13}C_3}$ (B) $\dfrac{12!}{5!\,3!\,2!} \times {^{13}P_3}$

(C) $\dfrac{13!}{5!\,3!\,2!\,3!}$ (D) None of these

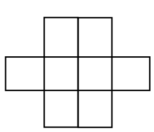
Sol. After removing three C from the given letters, we get 12 letters which can be arranged in a row in $\dfrac{12!}{5!\,3!\,2!}$ ways.

Now there are 13 places where we can write C. This can be done in $^{13}C_3$ ways.

Hence required no. of ways

$= \dfrac{12!}{5!\,3!\,2!} \times {^{13}C_3}$. **Ans. [A]**

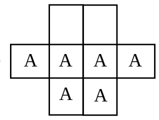
Ex.29 In the given figure of squares, 6 A's should be written in such a manner that every row contains at least one 'A', it is possible in number of ways is-

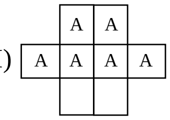


(A) 24 (B) 25 (C) 26 (D) 27

Sol. There are 8 squares and 6 'A' in given figure.

First we can put 6 'A' in these 8 squares by $^8C_6$ number of ways.

(I) 

(II) 

According to question, atleast one'A' should be included in each row. So after subtracting these two cases, number of ways are

$= ({^8C_6} - 2) = 28 - 2 = 26$ **Ans. [C]**

Ex.30 How many words can be formed taking 4 letters from the given 7 capital, 3 vowels and 5 consonants so that each word starts with a capital and contains atleast one vowel?

(A) 276 (B) 322

(C) 1932 (D) None of these

Sol. Here we have to fill 4 places. The first place can be filled in 7 ways as any one of 7 capital letters can be written at this place.

Now the remaining three places are to be filled in with the help of 3 vowels and 5 consonants so that at least one vowel is always included. The no. of ways $= {^8P_3} - {^5P_3}$

Therefore required no. of words $= 7({^8P_3} - {^5P_3})$

= 7 (336-60) = 7 x 276 = 1932

Ans. [C]

Ex. 31 If the letters of the word 'RACHIT' are arranged in all possible ways and these words are written out as in a dictionary, then the rank of this word is-

(A) 365 (B) 702

(C) 481 (D) None of these

Sol. The number of words beginning with A (i.e. in which A comes in first place) is $^5P_5 = 5!$.

Similarly number of words beginning with C is 5!, beginning with H is 5! and beginning with I is also 5!.

Now letters of R, four letters A, C, H, I can occur in 4 (5!) = 480 ways. Now word 'RACHIT' happens to be the first word beginning with R.

Therefore the rank of this word = 480 + 1 = 481.

Ans. [C]